From the Cauchy–Riemann equations there follows that $\alpha_j \equiv \alpha_j'$.

Then the function $\varphi_j('z)$ has the form $\varphi_j('z) = \alpha_j(\tilde{z}) z_{m+1} + \gamma_j(\tilde{z})$. By virtue of the fact that $\alpha_j(\tilde{z})$ is real and holomorphic, we conclude that $\alpha_j(\tilde{z}) \equiv \text{const} = a_{j,\, m+1}$. Applying successively the same arguments to the variables of the group $\tilde{z}$ we derive that the functions φ_j have the form

$$\varphi_j('z) = \sum_{s=m+1}^{k+l} a_{j,\, s} z_s + b_j.$$

Theorem 3 is proved. As mentioned above, from this Theorems 1 and 2 follow.

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PERTURBATIONS OF A BASIS IN A PAIR OF BANACH SPACES

A. N. Plichko and V. V. Shevchik UDC 517.982

We say that a Banach space X_1 is densely imbedded in a Banach space X_0 (we denote this by $X_1 \langle X_0$) if $X_1 \subset X_0$ as a linear subspace is dense there with respect to the norm $\|\,\|_0$, $X_1 \neq X_0$ and there exists a constant c such that $\|x\|_0 \leqslant c\|x\|_1$ for all $x \in X_1$. If the space X_1 is reflexive, then from $X_1 \langle X_0$ there follows that for the dual spaces we have $X_0^* \langle X_1^*$ and, moreover, the linear subspace $X_0^* \subset X_1^*$ has infinite defect. For a Hilbert space X we denote by J the canonical mapping X into $X^*: (Jx)(y) = \langle x, y\rangle$. It is an isometry of X onto X^*.

THEOREM. Let $X_1 \langle X_0$ be a pair of densely imbedded spaces and, moreover, assume that X_1 is a Hilbert space and X_0 has a basis $\{x_n\}_1^\infty$. Then there exists an orthogonal sequence $\{y_n\}_1^\infty \in X_1$ which is a basis in X_0 and its closed linear hull with respect to norm $\|\,\|_1$ has infinite defect in X_1.

Proof. Let $\varepsilon_n = 1/(2^n \|x_n\|_0 \|f_n\|_0)$ where $f_n \in X_0^*$ are functionals biorthogonal to x_n and let $J: X_1 \to X_1^*$ be the canonical mapping. We construct sequences $y_n \in X_1$ and $g_n, h_n \in X_1^*$ such that for each n we have:

$$1)\ h_i(y_j) = 0,\ i,\ j = 1, n,\quad h_n \notin \operatorname{lin}(X_0^*,\ (g_i,\ h_i)_1^{n-1});$$
$$2)\ g_i(y_j) = 0 \text{ for } i \neq j,\ i,\ j = 1, n,\ g_n = J(y_n),\ g_n \notin X_0^* + H_n;$$
$$3)\ \|x_n - y_n\| < \varepsilon_n,$$

where $H_n = \operatorname{lin}((h_i, g_i)_1^{n-1}, h_n)$, lin denoting the linear hull. We carry out the construction by induction. We select some functional $h_1 \in X_1^* \setminus X_0^*$. Then the annihilator $h_1^\perp \subset X_1$ is dense

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there with respect to the norm $\|\|_0$. Therefore, there exists an element $y_1 \in h_1^\perp$, $y_1 \notin J^{-1}(\mathrm{lin}(X_0^*, h_1))$ for which $\|x_1 - y_1\|_0 < \varepsilon_1$. We set $g_1 = Jy_1$. The validity of the conditions 1 – 3 for the elements y_1, h_1, g_1 is verified in a simple manner.

Assume that for $n-1$ the sequences have been constructed. We select some functional $h_n \in ((y_i)_1^{n-1})^\perp \subset X_1^*$, $h_n \notin \mathrm{lin}(X_0^*, (g_i, h_i)_1^{n-1})$. Since X_0^* has infinite defect in X_1^* this can be done. Since $H_n \cap X_0^* = 0$ the annihilator $H_n^\perp \subset X_1$ is dense there with respect to the norm $\|\|_0$. Therefore, there exists an element $y_n \in H_n^\perp$, $y_n \notin J^{-1}(X_0^* + H_n))$, for which $\|x_n - y_n\| < \varepsilon_n$. We set $g_n = Jy_n$. Only the first condition of part 2) has to be verified. Since $y_n \in H_n^\perp$ we have $g_i(y_n) = 0$ and $g_n(y_i) = \langle y_n, y_i\rangle = g_i(y_n) = 0$ for $i < n$. Condition 2) is verified.

From the well-known theorem on the stability of a basis [1, p. 70] and condition 3) there follows that $\{y_n\}_1^\infty$ is a basis in the space X_0. From the first part of property 2) there follows that the sequence y_n orthogonal in the Hilbert space X_1. According to the first part of condition 1) we have $l\ (y_n)_1^\infty \subset ((h_n)_1^\infty)^\perp$ and, according to the second part, the space lin $(h_n)_1^\infty$ is infinite-dimensional; therefore, the annihilator of $(h_n)_1^\infty$ has infinite defect.

<u>COROLLARY</u> [2]. There exists an orthonormal system which is a basis in $L_1[0,1]$ but not in $L_2[0,1]$.

In [3] one has proved a stronger result than the Corollary: for each interval $I \subset [1, 2]$ there exists an orthonormal system which forms a basis in $L_p[0,1]$ for all $p \in I$ and is not a basis in $L_q[0,1]$ for any $q \in [1, \infty] \setminus I \times (L_\infty[0,1] \overset{\text{def}}{=} C[0,1])$. Apparently, also this statement can be expressed in an abstract formulation, similar to the theorem in which instead of a pair of Banach spaces there occur interpolation families. Of course, the basis in the theorem can be replaced by other biorthogonal systems, stable with respect to small perturbations, for example, by an unconditional basis or by a Markushevich basis.

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