$$|x_{n_k k}(\alpha)| < 2/k \quad \text{for} \quad k \geqslant k_0.$$

Consequently, $\lim_{k\to\infty} x_{n_k k}(\alpha) = 0$. If $\lim_{r\to\infty} \varphi_r(\alpha) < \infty$, then $x_{ni}(\alpha) = 0$ for $i \in \mathbf{N}$, $n \geqslant \sup_r \varphi_r(\alpha)$. Hence in this case too $\lim_{k\to\infty} x_{n_k k} = 0$. The theorem is proved.

THEOREM 4. The assertion of the existence of a countable family of functions $\{x_{nk}\}$, defined on the set A of cardinality less than $\mathfrak{c}$, satisfying (2) and such that for any increasing sequence $\{n_k\}$ (3) does not hold, is consistent with the system ZFC.

Proof. The construction of such a family of fucntions is carried out in the Kunen model. Suppose we have strictly increasing sequences $\{n_\nu^\alpha\}$ ($\alpha \in A$, $|A| = \aleph_1 < \mathfrak{c} = \aleph_2$) of elements of a base of a free selective ultrafilter on N. We define functions x_{nk} by

$$x_{nk}(\alpha) = \begin{cases} 1, & n < n_1^\alpha, \\ k/\nu, & n_\nu^\alpha \leqslant n < n_{\nu+1}^\alpha, \end{cases} \quad k \in \mathbf{N}, \quad \alpha \in A.$$

Obviously (2) holds for these functions. The proof of the fact that (3) does not hold for any of the sequences $\{n_k\}$ is based on the property of selectivity of the ultrafilter and is analogous to the proof of Theorem 2. The theorem is proved.

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PROJECTIVE RESOLUTIONS, MARKUSHEVICH BASES, AND EQUIVALENT NORMS

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Let X be a Banach space and let X* be its dual. A subspace $F \subset X^*$ is said to be λ-norming, $\lambda > 0$, if the Dixmier characteristic

$$\inf \sup \{f(x)\colon f \in F, \|f\| \leqslant 1\} = \lambda,$$

where the infimum is taken over all $x \in X, \|x\| = 1$. When we are not interested in the value of λ, we speak simply of norming subspaces. A system (x_i, f_i), $x_i \in X$, $f_i \in X^*$ $i \in \mathcal{J}$ ($\mathcal{J}$ is some set) is said to be a Markushevich basis (an M-basis) if $f_i(x_j) = \delta_{ij}$ (δ is the Kronecker symbol), the norm-closed linear span $[x_i\colon i \in \mathcal{J}] = X$, and the subspace $F = [f_i\colon i \in \mathcal{J}] \subset X^*$, is total on X. If the subspace F is norming, then the M-basis will be said to be norming.

Notations. dens X is the weight, i.e., the smallest cardinality of an everywhere dense subset of the space X, ω is the first infinite and Ω is the first uncountable ordinal, $\bar{\bar{\alpha}}$ is the cardinality of the ordinal α, B(X) is the unit ball of the space X, $M^\perp$ is the annihilator and lin M is the linear span of the set M.

Let α be the first ordinal for the weight X. A collectively bounded set of projections $P_\beta\colon X \to X$, $\omega \leqslant \beta \leqslant \alpha$, is said to be a projective resolution of the identity if for any $\omega \leqslant \beta$, $\gamma \leqslant \alpha$: 1) $P_\beta P_\gamma = P_\gamma P_\beta = P_{\min(\gamma,\beta)}$; 2) dens $P_\beta X \leqslant \bar{\bar{\beta}}$; 3) $P_\beta X = [P_{\gamma+1}X\colon \gamma < \beta]$ and 4) $P_\alpha = I$ (the identity operator). From here it is easy to derive that for any $x \in X$ and ordinal β we have

$$\|(P_\beta - P_\gamma)x\| \to 0 \tag{1}$$

for $\gamma \to \beta$ in the order topology.

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In this note we prove that in spaces of weight there are not many projective resolutions (Theorem 1) and we give some corollaries of this result. We also give an example of a locally uniformly convex Banach space without an M-basis. This gives the answer to a question from [1] (Problem 20.8). It is interesting that the dual of a locally uniformly convex space has an M-basis [2].

LEMMA 1. Let P_β and Q_β, $\omega \leqslant \beta \leqslant \Omega$, be projective resolutions in the space X (of weight $\overline{\overline{\Omega}}$). For any $\beta_0 < \Omega$ there exists an ordinal $\beta_0 < \gamma < \Omega$ such that $P_\gamma X = Q_\gamma X$.

Proof. We construct a sequence $\beta_0 < \beta_n < \beta_{n+1} < \Omega$ such that for even n = 0, ∞ one has

$$Q_{\beta_n} X \subset P_{\beta_{n+1}} X \subset Q_{\beta_{n+2}} X.$$

Since dens $Q_{\beta_0} X \leqslant \overline{\overline{\beta}}_0 < \overline{\overline{\Omega}}$ and for any $x \in X$ there exists a sequence $x_n \in \bigcup_{\beta<\Omega} P_\beta X$, converging to x, it follows that there exists an ordinal $\beta_0 < \beta_1 < \Omega$ for which $Q_{\beta_0} X \subset P_{\beta_1} X$. Since dens $P_{\beta_1} X < \overline{\overline{\Omega}}$, there exists $\beta_1 < \beta_2 < \Omega$ with $P_{\beta_1} X \subset Q_{\beta_2} X$. For the same reason there exists $\beta_2 < \beta_3 < \Omega$ for which $Q_{\beta_2} X \subset P_{\beta_3} X$, etc. Then the ordinal $\gamma = \lim_n \beta_n$ satisfies the condition of the lemma.

LEMMA 2. Let P_β and Q_β, $\omega \leqslant \beta \leqslant \Omega$, be projective resolutions in the space X and let $X^* = \bigcup_{\beta<\Omega} P_\beta^* X^*$. Then for any $\beta < \Omega$ there exists an ordinal $\beta \leqslant \gamma < \alpha$ such that $Q_\beta^* X^* \subset P_\gamma^* X^*$.

Proof. We denote by dens*M the least cardinality of weak* subsets $M \subset X^*$, dense in M. The subspace $Q_\beta^* X^*$ is isomorphic to $Q_\beta X$ (in the natural duality) and, therefore, dens* $Q_\beta^* X^* \leqslant$ dens $Q_\beta X < \overline{\overline{\Omega}}$. Since $X^* = \bigcup_{\beta<\Omega} P_\beta^* X^*$, there exists an ordinal $\beta < \gamma < \Omega$, for which $Q_\beta^* X^* \subset P_\gamma^* X^*$.

THEOREM 1. Let P_β, Q_β, $\omega \leqslant \beta \leqslant \Omega$, be projective resolutions in space X and let

$$X^* = \bigcup_{\beta<\Omega} P_\beta^* X^*. \qquad (2)$$

Then for any $\beta_0 < \Omega$ there exists an ordinal $\beta_0 < \gamma < \Omega$, such that $P_\gamma = Q_\gamma$.

Proof. We construct a sequence $\beta_0 < \beta_n < \beta_{n+1} < \Omega$, $n = 1, \infty$, for which $Q_{\beta_n} X = P_{\beta_n} X$ for n = 3k + 1 and $Q_{\beta_n}^* X^* \subset P_{\beta_{n+1}}^* X^*$ for n = 3k + 2, k = 0, 1, 2, ... The ordinal β_1 exists according to Lemma 1, the ordinals β_2 and β_3 exist according to Lemma 2 (if one sets there $\beta = \beta_2$ and $\gamma = \beta_3$), the ordinal β_4 according to Lemma 1, etc. Then for $\gamma = \lim \beta_n$, $Q_\gamma X = P_\gamma X$ and $Q_\gamma^* X^* \subset P_\gamma^* X^*$. Consequently, $P_\gamma = Q_\gamma$.

COROLLARY 1. In a space of weight $\overline{\overline{\Omega}}$ the condition (2) is satisfied either for every projective resolutions or for none of them.

LEMMA 3. Let P_β, $\omega \leqslant \beta \leqslant \alpha$, be a projective resolutions in the space X, bounded by the number λ^{-1}. Then the characteristic of the subspace $H = \bigcup_{\beta<\alpha} P_\beta^* X^*$ is not less than λ. For any ordinal β, the weak* closure of the subspace $\bigcup_{\gamma<\beta} P_{\gamma+1}^* X^*$ coincides with $P_\beta^* X^*$.

Proof. Let $x \in X$ be an arbitrary element with $\| x \| = 1$ and let $f \in X^*$ be a functional for which $\| f \| = f(x) = 1$. Then, according to relation (1),

$$1 = \langle x, f \rangle = \lim_{\beta\to\alpha} \langle P_\beta x, f \rangle = \lim_{\beta\to\alpha} \langle x, P_\beta^* f \rangle.$$

From here,

$$\sup \{h(x) : h \in B(H)\} \geqslant \lim_{\beta\to\alpha} \langle x, \lambda P_\beta^* f \rangle = \lambda.$$

The first part of the lemma is proved. If for some $y \in X$ there exists a functional $f \in P_\beta^* X^*$, such that $f(y) \neq 0$, while $\bigcup_{\gamma<\beta} P_{\gamma+1}^* X^* \subset y^\perp$, then $x = P_\beta y$ has the same property. Indeed, $\langle P_\beta y, f \rangle = \langle y, P_\beta^* f \rangle = \langle y, f \rangle \neq 0$ and for any $g \in \bigcup_{\gamma<\beta} P_{\gamma+1}^* X^*$ we have

$$\langle P_\beta y, g \rangle = \langle y, P_\beta^* g \rangle = \langle y, g \rangle = 0.$$

From here we obtain the contradicting relation

$$0 \neq \langle x, f \rangle = \lim_{\gamma+1\to\beta} \langle P_{\gamma+1} x, f \rangle = \lim_{\gamma+1\to\beta} \langle x, P_{\gamma+1}^* f \rangle = 0.$$

We recall that a Banach space X has a weak* angelic dual [3] if the weak* sequential closures of each bounded subset of X* coincide. In particular, any WCG-space (a space which is the closed linear span of its weak compact subset) has a weak* angelic dual.

COROLLARY 2. Let P_β, $\omega \leqslant \beta \leqslant \Omega$ be a projective resolution in a Banach space X with a weak* angelic dual. Then the subspace $H = \bigcup_{\beta<\Omega} P_\beta^* X^*$ coincides with X*.

Proof. From Lemma 3 it follows that the subspace H is norming. This means that for some number r the weak* closure of the ball rB(H) contains the ball B(X*) [4]. Since X* is weak* angelic, for any element $f \in X^*$ there exists a sequence $f_n \in P_{\beta_n}^* X^*$, which is weakly* convergent to f. We set $\beta = \lim_n \beta_n$. Then $\beta < \Omega$ and, since the subspace $P_\beta^* X^* \subset X^*$ is weak* closed, we have $f \in P_\beta^* X^* \subset H$.

As it is known, every WCG-space has a projective resolution with norms $\| P_\beta \| = 1$. Moreover, $\| I - P_\beta \| \leqslant 2$. We give an example when this bound cannot be improved.

Example. Let $c(\Omega)$ be the space of real functions on the space of ordinals $[1, \Omega]$, satisfying the condition: for each $x \in c(\Omega)$ and each $\varepsilon > 0$ there exists a constant a such that the inequality $|x(\beta) - a| > \varepsilon$ is satisfied only for a finite number of ordinals β with the supremum norm. Let x_0 be the function identically equal to unity and let x_β, $1 \leqslant \beta < \Omega$, be the standard unit vectors, i.e., functions equal to unity at the point β and to zero at the remaining points. The set $(x_\beta: 0 \leqslant \beta < \Omega)$ is weakly compact and, therefore, $c(\Omega)$ is a WCG-space. Let Q_β, $\omega \leqslant \beta < \Omega$, be the projection operators onto $X_\beta = [x_\gamma: 0 \leqslant \gamma < \beta]$ parallel to $X^\beta = [x_\gamma: \beta \leqslant \gamma < \Omega]$. It is easy to see that they form a projective resolution. If $x \in \operatorname{lin}(x_\gamma: \gamma < \beta)$, $y \in \operatorname{lin}(x_\gamma: \gamma \geqslant \beta)$, then, considering separately the cases when the function $|x(\gamma)|$ attains its maximum on an ordinal $<\beta$ and $\geqslant \beta$ (in the latter case it attains the maximum on any ordinal $\geqslant \beta$), we obtain that $\| x \| \leqslant \| x - y \|$. Consequently, $\| Q_\beta \| = 1/\inf\{\| x - y \|: x \in X_\beta, \| x \| = 1, y \in X^\beta\} = 1$. In addition, $\| x_\gamma - x_0/2 \| = 1/2$ so that $\| I - Q_\beta \| = 2$.

Let P_β be some projective resolution in the space $c(\Omega)$. Then, according to Theorem 1 and Corollary 2, there exists an ordinal $\gamma < \Omega$ such that $P_\gamma = Q_\gamma$ and, consequently, $\| I - P_\gamma \| = 2$.

We give now an example of a space of weight $\bar{\bar{\Omega}}$ with a projective resolution for which condition (2) does not hold. We denote by $C_0(\Omega)$ the space of order continuous numerical functions on the segment $[1, \Omega]$, convergent to zero for $\beta \to \Omega$ with the supremum norm. For an ordinal $1 \leqslant \beta < \Omega$ we set $x_\beta(\gamma) = 1$ for $1 \leqslant \gamma \leqslant \beta$ and $x_\beta(\gamma) = 0$ for $\gamma > \beta$. The biorthogonal functionals have the form $f_\beta(x) = x(\beta) - x(\beta + 1)$.

THEOREM 2. The space $X = C_0^-(\Omega)$ has the following properties:

1) the system (x_β, f_β) forms an M-basis;

2) the projections P_β onto the subspace $[x_\gamma: \gamma < \beta]$, parallel to $[x_\gamma: \gamma \geqslant \beta]$, exist and form a projective resolution with norms $\| P_\beta \| = 2$;

3) the ball B(X*) is weak* sequential compact;

4) X* is not weak* angelic, more exactly, the subspace $H = \bigcup_{\beta<\Omega} P_\beta^* X^*$ is weak* sequential compact, 1/2-norming, and does not coincide with X*.

Proof. The conditions 1), 2) can be easily verified and have been proved in [5]. For the verification of the condition 3) we denote by X_β the functions from X, vanishing outside the segment $[1, \beta]$ and by X^β the functions that are equal to zero on $[1, \beta]$. Then $X = X_\beta \oplus X^\beta$. It is easy to see that for any functional $f \in X^*$ there exists an ordinal $\beta < \Omega$ such that $f \in (X^\beta)^\perp$. Indeed, if f does not belong to any of the annihilators $(X^\beta)^\perp$, $\beta < \Omega$, then this means that there exists a countable set $(x_\gamma) \subset X$ for which $\operatorname{supp} x_\gamma \cap \operatorname{supp} x_\delta = \varnothing$ for $\delta \neq \gamma$ and $f(x_\gamma) > \varepsilon$, where $\operatorname{supp} x = \{\gamma: x(\gamma) \neq 0\}$. This contradicts the boundedness of the functional f. Assume now that (f_n) is a sequence from the ball of X*. Then there exists $\beta < \Omega$ such that $(f_n) \subset (X^\beta)^\perp$. The annihilator $(X^\beta)^\perp$ is isomorphic to the conjugate of the separable space X_β (in the natural duality). Since the ball of the dual of a separable space is weakly* sequential compact, one can select from f_n a weakly* convergent subsequence.

We verify condition 4). According to condition 2 and Lemma 3, the subspace H is 1/2-norming. Obviously, it is weak* sequential closed. But the functional $g(x) = x(1)$ does not belong to H since it does not vanish on any subspace $(I - P_\beta)X$.

LEMMA 4. Assume that the space X has a weak* angelic dual. Any of its subspaces and factor-spaces has a weak* angelic dual.

Proof. The second part of the assertion is obvious. Let Y be a subspace of X and let G be a bounded (by unity) subset of $Y^* = X^*/Y^\perp$. For some number $b > 1$ we set $A = \varphi^{-1} G \cap bB(X^*)$, where φ is the quotient mapping $X^* \to X^*/Y^\perp$. We denote by $\bar{A}$ and $\bar{G}$ the weak* closures of the corresponding sets. Let $g \in \bar{G}$. This means that there exists a net $g_\alpha \in G$, weakly* convergent to g. Since the set $\bar{A}$ is weak* compact and $\varphi(A) = G$, from the net of any preimages $\varphi^{-1}g_\alpha \in A$ one can select a subnet, weakly* convergent to an element $a \in \bar{A}$. Since the mapping φ is weak* continuous, we have $\varphi(a) = g$. The space X* is weak* angelic and, therefore, there exists a subsequence $a_n \in A$, weakly* convergent to a. Then $\varphi(a_n) \xrightarrow{w^*} \varphi(a) = g$, and, consequently, Y* is angelic in the weak* topology.

We recall that the norm of a Banach space is said to be locally uniformly convex if for any $x \in X$, $\| x \| = 1$, from the relations $\| y_n \| = 1$, $\| y_n + x \| \to 2$ there follows $\| y_n - x \| \to 0$. We consider the conjugate $\mathcal{J}T^*$ of the James tree $\mathcal{J}T$. It has the following properties [6]:

1) $\mathcal{J}T$ is separable and does not contain subspaces isomorphic to l_1, while $\mathcal{J}T^*$ is nonseparable;

2) There exists a separable subspace $Y \subset \mathcal{J}T^*$ for which $\mathcal{J}T^*/Y$ is isomorphic to a Hilbert space.

We denote by X a subspace of $\mathcal{J}T^*$ of weight $\bar{\bar{\Omega}}$ containing Y.

THEOREM 3. The subspace X has an equivalent locally uniformly convex norm and a weak* angelic dual, but it has neither a projective resolution nor an M-basis.

Proof. From conditions 1), 2) and Theorem 3.1.1 of [5] there follows that the space X has an equivalent locally uniformly convex norm. According to condition 1), $\mathcal{J}T^{**}$ is weak* angelic [7], therefore (Lemma 4) X* is weak* angelic. In the same way as it has been proved in [8], for WCG-spaces one can show that if X* is weak* angelic, then any M-basis (x_i, f_i), $i \in \mathcal{I}$, in the space X is countably 1-norming i.e., the subspace $F \subset X^*$ consisting of elements f for which the set $\{i \in \mathcal{I}\colon f(x_i) \neq 0\}$ is countable, is 1-norming. By Theorem 1 of [8], from such an M-basis in the space X one can construct a projective resolution of the identity, P_β, $\omega \leqslant \beta \leqslant \Omega$. Therefore, there exists an ordinal $\beta < \Omega$ such that $Y \subset P_\beta X$. The subspace $P_\beta X$ is separable and complemented in X. But it is actually proved in [9] that if a nonseparable subspace of X has a reflexive quotient with respect to a separable subspace Y and the weak* separable X*, then for the subspace Y there is no separable subspace $Y \subset Y' \subset$ X, complemented in X. Consequently, H has neither an M-basis nor a projective resolution.

Let X be a Banach space and let F be a total subspace of X*. In analogy with [10], the norm of the space X will be called a Kadec F-norm if on the sphere S(X) it coincides with the weak topology $\sigma(X, F)$. This is equivalent to the fact that for each $x \in S(X)$ and $\varepsilon > 0$ the point x does not belong to the $\sigma(X, F)$-closure of the set $B(X) \setminus B(x, \varepsilon)$, where $B(x, \varepsilon)$ is the ball with center at x and radius ε.

THEOREM 4. If dens F < dens X, then the norm of the space X is not a Kadec F-norm.

Proof. Let $Y \subset S(X)$ be a set of cardinality dens X with $\| y - y' \| > \varepsilon$ for some $\varepsilon > 0$ and for all $y, y' \in Y$, $y \neq y'$. Let G be a subset of F of cardinality dens F and dense with respect to the norm. We assume that $\| \ \|$ is a Kadec F-norm. Then for each point $y \in Y$ there exists a finite sequence $g_1, \ldots, g_n \in G$ and rational numbers $r_1, \ldots, r_n$ such that the set $\{x \in Y\colon g_1(x) > r_1, \ldots, g_n(x) > r_n\} = \{y\}$. Consequently, to each point $y \in Y$ there corresponds a finite sequence $\{(g_1, r_1), \ldots, (g_n, r_n)\}$, and to different $y, y' \in Y$ there correspond different sequences $\{(g_1, r_1), \ldots, (g_n, r_n)\} \neq \{(g_1', r_1'), \ldots, (g_{n'}', r_{n'}')\}$. Thus, the set of all finite collections $\{(g_1, r_1), \ldots, (g_n, r_n)\}$ (of cardinality equal to dens F) has a cardinality of least dens X. Contradiction.

Remark. If the subspace F is separable, then $\| \ \|$ is a Kadec F-norm if and only if it has the H_F-property, i.e., on the sphere S(X) the $\sigma(X, F)$-convergence coincides with the convergence in the norm. Indeed, the space X can be imbedded in a natural manner into F* and the ball $B(X) \subset B(F^*)$. Since the restriction of the topology $\sigma(F^*, F)$ to the ball $B(F^*)$ is metrizable, it follows that the $\sigma(X, F)$-closure and the $\sigma(X, F)$-sequential closure on the sphere S(X) coincide. In [5] one has given some examples of Banach spaces X with the H_{X^*}-property and with a norming subspace $F \subset X^*$, for which X does not possess the H_F-property for any equivalent norm. Making use of Theorem 4, the number of such examples can be extended. Thus, the space $X = l_1[0, 1]$ has the H_{X^*}-property but for the separable norming subspace $F = C[0, 1] \subset l_\infty[0, 1] = X^*$ there is no equivalent norm on X with the H_F-property.

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REPRESENTATION OF FUNCTIONS IN $P_{\aleph_0}$ BY SUPERPOSITIONS OF ONE-PLACE FUNCTIONS AND ADDITION

I. G. Perfil'eva

In the theory of functional systems the representation of the functions of a system by superpositions of functions of a complete subsystem is part of the more general problem of completeness. As examples can serve the representation of Boolean functions in perfect disjunctive normal form and the corresponding representation of functions of many valued logic, or the representation of functions of p-valued logic by modulo-p polynomials (p being a prime), etc. (see [1]).

However, the problem of representation is also of intrinsic interest, due to the fact that "composite" many place functions can be expressed in terms of "simple" one-place and two-place functions. Thus, without the use of the concept of completeness, a result has been obtained concerning the representation of any continuous function by a superposition of continuous fuctions of a variable and addition [2]. A similar result is obtained below for functions of countable valued logic.

Let us list all the necessary definitions and notations [1, 3]: $E^{\aleph_0} = \{0, 1, 2, \ldots\}$ is the set of all nonnegative integers; $P_{\aleph_0}$ is a countable-valued logic, i.e., the set of all n-place functions $(n \geqslant 1)$ defined on the set $(E^{\aleph_0})^n$ and that take their values on the set $E^{\aleph_0}$; I_0 is the set of all one-place functions of countable valued logic that have only infinite level sets and that take each value in $E^{\aleph_0}$; $C(m)$ is the set of all one-place functions of countable valued logic that take precisely m values, $m \geqslant 1$;

$$R = C(1) \cup C(2) \cup I_0 \cup \{x + y\}.$$

The principal result of this paper is expressed by the following

THEOREM. In a countable-valued logic any function $g(x_1, \ldots, x_n)$, $n \geqslant 1$, can be represented by a superposition of the form

$$g(x_1, \ldots, x_n) = r^{g_0}\left(r^0\left(\textstyle\sum_{i=1}^{n} r^{g_i}\left(r^0(x_i)\right)\right)\right), \quad n \geqslant 2, \tag{1}$$

or for n = 1,

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